Efficient Integral Equation Formulations for Admittance or Impedance Representation of Planar Waveguide Junctions

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Abstract

In this paper we describe very efficient *impedance* and *admittance* Integral Equation (IE) formulations for the study of passive microwave devices composed of cascaded uniform waveguide sections. The formulation leads directly to reduced multimode matrix representations that only involve a small number of accessible, or interacting, modes. In addition to theoretical results, a performance comparison is also discussed which clearly demonstrates the improvement achieved.

I Introduction

The evolution of telecom satellites imposes ever decreasing manufacturing times and costs. There is, therefore, a constant demand for advanced CAD tools [1]. Among the many formulations developed, the admittance matrix has been recently shown to be both very accurate and very efficient [2]. The computational effort which is required, following [2], is composed of two parts. The first is frequency independent and consists of computing the *coupling integrals*. The second consists of inverting a linear system. What determines the computational efficiency of a CAD tool is essentially the dimension of this system. This, in turn, depends essentially on two factors: the number of modes required to represent the electromagnetic field at each junction, and the number of modes which must be used to represent the interactions. The number of interacting, or accessible, modes is substantially smaller than the number of modes required to represent the fields (localized modes).

To exploit this difference, we need a more efficient solution of the field problem. An integral equation approach has been recently developed [3], which could, in principle, be used. However, the procedure in [3] does not effectively separate the accessible and localized modes.

In this paper, we first discuss two integral equation formulations which give directly the multimode impedance or admittance matrix representation of a waveguide junction in terms of a *finite set of accessible modes only.* We then show how *the frequency dependence* can be effectively *extracted* from kernel of the integral equation thus further increasing the computational efficiency. Finally, we also present application results obtained for a typical structure clearly showing the improvement achieved.

II Impedance Integral Equation

The problem under investigation is the junction between two arbitrary waveguides (Fig. 1). The first step in the Impedance Integral Equation formulation is to impose the boundary conditions at the junction, namely

$$\sum_{n=1}^{\infty} I_n^{(1)} \mathbf{h}_n^{(1)} = \sum_{n=1}^{\infty} I_n^{(2)} \mathbf{h}_n^{(2)} \qquad (1)$$

where the index n covers both $TE_{m,n}$ and $TM_{m,n}$ modes and where the superscripts (1) and (2) refer to regions (1) and (2) in Fig. 1. We then separate the accessible from the localized modes, by writing

$$\sum_{n=1}^{N^{(1)}} I_n^{(1)} \mathbf{h}_n^{(1)} - \sum_{n=1}^{N^{(2)}} I_n^{(2)} \mathbf{h}_n^{(2)} = \sum_{n=N^{(1)}+1}^{\infty} V_n^{(1)} Y_n^{(1)} \mathbf{h}_n^{(1)} + \sum_{n=N^{(2)}+1}^{\infty} V_n^{(2)} Y_n^{(2)} \mathbf{h}_n^{(2)} \quad (2)$$

where $Y_n^{(\delta)}$ is the modal admittance. We now recall that

$$V_n^{(\delta)} = \int_{cs} (\mathbf{z}_o \times \mathbf{E}) \cdot \mathbf{h}_n^{(\delta)*} ds' \qquad (3)$$

where the electric field in the aperture $(\mathbf{z}_o \times \mathbf{E})$ can now be written as a linear combination of the *incident* accessible modes only

$$(\mathbf{z}_o \times \mathbf{E}) = \sum_{n=1}^{N^{(1)}} \bar{I}_n^{(1)} \mathbf{M}_n^{(1)} - \sum_{n=1}^{N^{(2)}} \bar{I}_n^{(2)} \mathbf{M}_n^{(2)} \quad (4)$$

This expression can now be used in (3) and (2) so that we finally obtain the integral equation

$$\mathbf{h}_n^{(\delta)} = \int_{cs} \mathbf{M}_n^{(\delta)} \cdot \left[\sum_{m=1}^{N^{(1)}} \hat{Y}_m^{(1)} \mathbf{h}_m^{(1)} \mathbf{h}_m^{(1)*} + \sum_{m=N^{(1)}+1}^{\infty} Y_m^{(1)} \mathbf{h}_m^{(1)} \mathbf{h}_m^{(1)*} + \sum_{m=1}^{N^{(2)}} \hat{Y}_m^{(2)} \mathbf{h}_m^{(2)} \mathbf{h}_m^{(2)*} + \sum_{m=N^{(2)}+1}^{\infty} Y_n^{(2)} \mathbf{h}_m^{(2)} \mathbf{h}_m^{(2)*} \right] ds' \quad (5)$$

To conclude, we now recall (3) and (4), and write

$$V_m^{(\gamma)} = \sum_{n=1}^{N^{(1)}} \bar{I}_n^{(1)} \mathbf{Z}_{m,n}^{(\gamma,1)} - \sum_{n=1}^{N^{(2)}} \bar{I}_n^{(2)} \mathbf{Z}_{m,n}^{(\gamma,2)} \qquad (6)$$

where

$$\mathbf{Z}_{m,n}^{(\gamma,\delta)} = \int_{cs} \mathbf{M}_n^{(\delta)} \cdot \mathbf{h}_m^{(\gamma)*} ds' \qquad (7)$$

Equations (5) and (7) complete the formal solution of the problem in Fig. 1 in terms of the *finite* impedance multimode equivalent network shown in Fig. 2.

III Admittance Integral Equation

The same general procedure described in the previous section can also be used to formulate a multimode admittance matrix representation using as a starting point the continuity of the electric field in the aperture. The relevant equations then become

$$\mathbf{e}_n^{(\delta)} = \int_{cs(1)} \mathbf{M}_n^{(\delta)} \cdot \left[\sum_{m=1}^{N^{(1)}} \hat{Z}_m^{(1)} \mathbf{e}_m^{(1)} \mathbf{e}_m^{(1)*} + \sum_{m=N^{(1)}+1}^{\infty} Z_m^{(1)} \mathbf{e}_m^{(1)} \mathbf{e}_m^{(1)*} \right] ds' + \int_{cs(2)} \mathbf{M}_n^{(\delta)} \cdot \left[\sum_{m=1}^{N^{(2)}} \hat{Z}_m^{(2)} \mathbf{e}_m^{(2)} \mathbf{e}_m^{(2)*} + \sum_{m=N^{(2)}+1}^{\infty} Z_n^{(2)} \mathbf{e}_m^{(2)} \mathbf{e}_m^{(2)*} \right] ds' \quad (8)$$

Where $\mathbf{M}_n^{(\delta)}$ is now related to the unknown magnetic field in the aperture

$$-(\mathbf{z}_o \times \mathbf{H}) = \sum_{n=1}^{N^{(1)}} \bar{V}_n^{(1)} \mathbf{M}_n^{(1)} - \sum_{n=1}^{N^{(2)}} \bar{V}_n^{(2)} \mathbf{M}_n^{(2)} \quad (9)$$

The above expression can now be used to obtain

$$\mathbf{Y}_{m,n}^{(\gamma,\delta)} = \int_{cs(\delta)} \mathbf{M}_n^{(\delta)} \cdot \mathbf{e}_m^{(\gamma)*} ds' \qquad (10)$$

This concludes the admittance formulation, giving the network shown in Fig. 2.

IV Extraction of the frequency dependence

Before using the networks in Fig. 2, the integral equations in (5) and (8) must be solved using, for instance, the method of moments. To do so, it is convenient to extract first the frequency dependence from the kernel of the integral equations. To illustrate this point, we first write (impedance case)

$$\mathbf{K}(s,s') = \left[\sum_{m=1}^{N^{(1)}} \hat{Y}_m^{(1)}\mathbf{h}_m^{(1)}\mathbf{h}_m^{(1)*} + \sum_{m=N^{(1)}+1}^{\infty} Y_m^{(1)}\mathbf{h}_m^{(1)}\mathbf{h}_m^{(1)*} + \sum_{m=1}^{N^{(2)}} \hat{Y}_m^{(2)}\mathbf{h}_m^{(2)}\mathbf{h}_m^{(2)*} + \sum_{m=N^{(2)}+1}^{\infty} Y_n^{(2)}\mathbf{h}_m^{(2)}\mathbf{h}_m^{(2)*}\right] \quad (11)$$

we then add and subtract the complement to infinity of the static terms, obtaining

$$\mathbf{K}(s,s') = \left[\sum_{m=1}^{\infty} \hat{Y}_m^{(1)}\mathbf{h}_m^{(1)}\mathbf{h}_m^{(1)*} + \sum_{m=1}^{\infty} \hat{Y}_m^{(2)}\mathbf{h}_m^{(2)}\mathbf{h}_m^{(2)*} - \sum_{m=N^{(1)}+1}^{\infty} \hat{Y}_m^{(1)}\left(1-\frac{Y_m^{(1)}}{\hat{Y}_m^{(1)}}\right)\mathbf{h}_m^{(1)}\mathbf{h}_m^{(1)*} - \sum_{m=N^{(2)}+1}^{\infty} \hat{Y}_m^{(2)}\left(1-\frac{Y_m^{(2)}}{\hat{Y}_m^{(2)}}\right)\mathbf{h}_m^{(2)}\mathbf{h}_m^{(2)*}\right] \quad (12)$$

The first two terms of (12) are now static, while for the other two

$$\lim_{m\to\infty}\left(1-\frac{Y_m^{(\delta)}}{\hat{Y}_m^{(\delta)}}\right) = 0 \quad (13)$$

so that

$$\left(1-\frac{Y_m^{(\delta)}}{\hat{Y}_m^{(\delta)}}\right) \simeq \sum_{p=1}^{P} B_{(2p)}^{(\delta)}(m)\left(\frac{k_o}{k_{t,m}^{(\delta)}}\right)^{(2p)} \quad (14)$$

Collecting all together:

$$\mathbf{K}(s,s') = \hat{\mathbf{K}}(s,s') - \sum_{p=1}^{P}(k_o)^{2p}\hat{\hat{\mathbf{K}}}_{2p}(s,s') \quad (15)$$

where

$$\hat{\mathbf{K}}(s,s') = \sum_{m=1}^{\infty} \hat{Y}_m^{(1)}\mathbf{h}_m^{(1)}\mathbf{h}_m^{(1)*} + \sum_{m=1}^{\infty} \hat{Y}_m^{(2)}\mathbf{h}_m^{(2)}\mathbf{h}_m^{(2)*} \quad (16)$$

$$\hat{\hat{\mathbf{K}}}_p(s,s') = \sum_{m=N^{(1)}+1}^{\infty} \frac{B_{(p)}^{(1)}(m)}{\left(k_{t,m}^{(1)}\right)^p}\hat{Y}_m^{(1)}\mathbf{h}_m^{(1)}\mathbf{h}_m^{(1)*} + \sum_{m=N^{(2)}+1}^{\infty} \frac{B_{(p)}^{(2)}(m)}{\left(k_{t,m}^{(2)}\right)^p}\hat{Y}_m^{(2)}\mathbf{h}_m^{(2)}\mathbf{h}_m^{(2)*} \quad (17)$$

Where the expressions for the coefficients $B_{(p)}^{(\delta)}(m)$ can be easily computed. All of the summations involved in the computation of the kernel are now static and can be carried out outside the frequency loop.

V Application example

As an application example, we discuss the analysis of the six pole filter (Fig. 3). The filter has been first analyzed following the approach described in [2], obtaining the results shown in Fig. 4. For these computations we have used 200 modes to describe the fields and 10 modes to describe the interactions. The required computation time was 37 minutes using 7 IBM RS6000 platforms in a parallel configuration, for 200 points in frequency. The serial computation time for the same structure on the same platform was 129 minutes.

In Figure 5 we show the results obtained using both the admittance and the impedance matrix formulations. The computations have been performed using again 10 accessible modes, 200 basis and test functions, and 500 terms in the summations of the kernels. The time required is 17 min for both formulations using again the IBM platform.

Although the impedance and the admittance formulations produce very similar final results, they do not exhibit the same convergency behavior. In particular, it appears that the impedance formulation is numerically more robust. We can, in fact, decrease the number of accessible modes to 3, the number of test and basis functions to 23, and the terms summed in the series to 400, to obtain the results shown in Fig. 6. The computation time is reduced to 27 seconds for 200 point in frequency.

VI Conclusion

In this paper we describe impedance and admittance integral equation formulations for arbitrary waveguide steps. The formulations described give directly the multimode network representations in terms of a reduced set of accessible modes only. Furthermore, we show how the frequency dependence can be essentially extracted from the kernel of the integral equations so that very efficient CAD tools can be implemented. In addition to theory, the application to a typical microwave filter structure is also discussed clearly indicating orders of magnitude improvements in the computation time.

VII Acknowledgement

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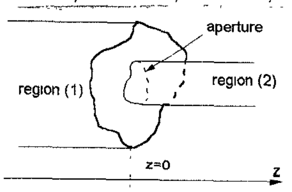

Figure 1

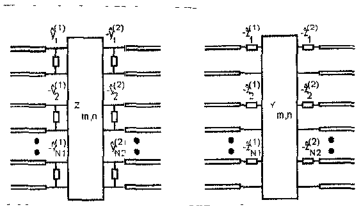

Figure 2: Multimode impedance and admittance network representations for the junction in Fig. 1.

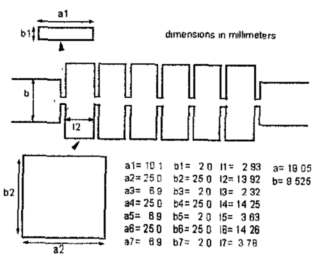

Figure 3: Microwave filter structure

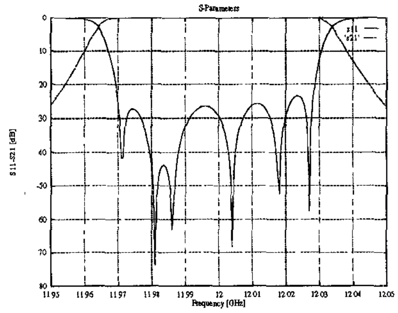

Figure 4: Simulated response of the filter in Fig. 3 obtained following the approach described in [2]

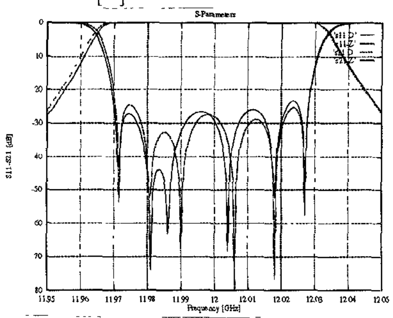

Figure 5: Simulated response of the filter in Fig. 3 obtained using the networks described in this paper.

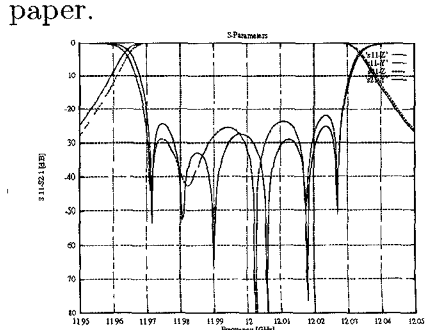

Figure 6: Same as in Fig. 5, but using only the impedance formulation with reduced computation accuracy. The results identified with the letter D are the same as in Fig. 4.